FRANCE STRATÉGIE
ÉVALUER. ANTICIPER. DÉBATTRE. PROPOSER.

Guide to the socioeconomic evaluation of public investments in France

Operational supplement I

REVISION OF THE DISCOUNT RATE[1]

Written by Jincheng Ni, France Stratégie, and Joël Maurice, member of the Committee of Experts[2]

under the authority of the Committee of Experts on Methods for Socioeconomic Evaluation of Public investments, chaired by Roger Guesnerie, professor at the Collège de France and Honorary President of Paris School of Economics

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[1] This operational supplement constitutes the instructions for public project owners. It supplements the *Guide to the socioeconomic evaluation of public investment* (France Stratégie/General Treasury Directorate, 2017), available online.

[2] With the help of Émile Quinet, Anne Jaubertie, Pierre-Adrien Collet, Simon Martin and Bérengère Mesqui. The Committee of Experts thanked the working group on discount rates (Émile Quinet, Joël Maurice, Michel Massoni, David Meunier, Jean-Jacques Becker, Alain Trognon and Jincheng Ni) and the two trainees (Victoria Grimaud and Asmae Marhraoui). He thanked all its members who participated in the discussions and deliberations.

Summary

The Quinet report (2013)[3] linked risk and discount rate with the formula:

$\rho = r_f + \beta.\varphi$, where $\rho = 2.5\% + \beta.2\%$ until 2070 and $\rho = 1.5\% + \beta.\ 3\%$ beyond.

At the request of the Commissioner General of France Stratégie and the Secretary General for Investment, the Committee of Experts on Methods for Socioeconomic Evaluation of Public investments has been working since 2017 to update the recommendations of the Quinet report on risk and discount rates.

In the opinion deliberated on 24 June 2021, the Committee makes the following recommendations:

- Retain a discount rate equal to **ρ = 1.2% + β. 2%** for the period from 2021 to 2070, which assumes knowing **β**.
- If β is unknown, it is proposed to proceed as if β were equal to 1. **The discount rate ρ to be used is then 3.2%,** the rate of change in the mathematical expectation of benefit v is 1.6%.
- If β is known, the rate of change in the mathematical expectation of benefit v is worth $\beta.1.15\ \% + \frac{\beta^2}{2}.0.9\ \%$.

A group is currently working on the calculation of certain β sectors. These will be published upon validation by the Committee of Experts.

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[3] French General Commission for Strategy and Foresight (2013), *Cost benefit assessment of public investments* , report of the mission chaired by Émile Quinet, September.

Introduction

Socioeconomic (cost benefit) assessment is one of the main tools available to public project owners to assess the relevance of an investment project for the community. It evaluates the creation of value for the community allowed by the project, by comparing the well-being gains and costs that this project brings to the national community throughout its lifetime. The calculation of the socioeconomic net present value (SE-NPV) makes it possible to determine whether, at the time the investment is involved, the value of the benefits for the community exceeds the value of the costs incurred to obtain them, and also to present elements of comparison between projects thus evaluated homogeneously.

The projects under review typically have costs and benefits spread over time: schematically, an expensive initial investment reports pays off over its lifetime, during which expenditure will have to be incurred, for example, to operate, maintain and renew an installed structure. It is therefore necessary to compare benefits (market and non-market benefits such as time savings, value of life, climate and environmental benefits, value of the diploma, etc.) and costs that occur at different times. The discount rate gives present value to the economic and financial flows measured in euros spent or earned in different years. The difference between the discounted benefits and costs is the socioeconomic net present value of the project.

The discount rate plays a central role, since it commands the trade-off between the present and the future: a high rate weakens the weight of the future, whether it is the near future or the more distant future of future generations.

The reference public discount rate has already been revised several times since 1980. This note tracks the history and presents the current revision and recommendations.

The 1980s

"In the 1960s, Pierre Massé, then Commissioner General of the Plan, was the first, after discussions with Edmond Malinvaud and Marcel Boiteux, to set the value of what he called the implicit interest rate of the economy: he was proud of it. This concept, which we currently call the discount rate, reflects the relative price we attach to the present and sets the limit that we are ready to grant for the future. This rate makes it possible to compare economic values over time"[4].

In 1985, the French Planning Commission (Commissariat général au plan) set a discount rate of 8% for the economy, based on the estimated marginal return on industrial capital of 6%, and added a 2% risk premium to ensure that the public investments selected would not replace private investments with a higher return.

This 8% discount rate remained unchanged until the mid-2000s, when it became apparent that this value, which was very high compared to the value of real interest rates at the time, mechanically led to too much overshadowing of what would happen in the distant future. Its revision was decided following the audit report[5] on major transport infrastructures published by the General Inspectorate of Finances and the General Council of Bridges and Roads in early 2003, which called into question the rejection of a large number of long-term investment

[4] French General Commission for Strategy and Foresight (2013), *L'Évaluation socioéconomique des investissements publics [The Socioeconomic evaluation of public investment], op. cit.* Extract from the preface written by Jean-Pisani Ferry.

[5] General Finance Inspection/General Council of Bridges and Roads (2003), *Rapport d'audit sur les grands projets d'infrastructures [Audit Report on Major Infrastructure Projects]*, February.

projects, for which the 8% discount rate resulted in negative SE- NPV.

The Lebègue Report (2005)

In 2005, the Lebègue Report (2005)[6] proposed a new discount rate, known as risk-free, for the economy. He therefore recommended a rate of 4% over 30 years and decreasing to 2% for very long timeframes beyond 30 years. The justification for this lowered rate was based on the logic of intertemporal arbitrations associated with the Ramsey formula (1928)[7]. The analysis did not include an in-depth examination of risk of the project, other than to reiterate that it should be explicitly taken into account in the calculation of the SE-NPV of each project.

The Gollier report (2011)

The Gollier report (2011)[8] presented a very comprehensive analysis of the risks to which a public project is subject and the ways in which they should be dealt with. The public economic calculation could not ignore the non-diversifiable risks, against which States cannot protect themselves and for which there is no possible pooling.

This report recommends integrating systemic risks into the economic assessment of investment projects. To do this, he recommends introducing a so-called systemic risk premium, which will be positive (negative) when the fundamentals of the project are positively (negatively) correlated with economic activity.

Indeed, if the risks are correlated to economic growth, a risk premium must be taken into account so as to penalise projects that increase the collective risk most and give a bonus to projects that insure the community against this collective risk.

The É. Quinet report (2013)

The É. Quinet report (2013)[9], following on from the Gollier report, provided details of the new discounting system and calibrated the discount rate ρ, which incorporates a reference rate and a risk premium to reflect the effect of non-diversifiable risk.

The reference rate, noted below as r_f is decisive in determining the level of transfers to be made by present generations (trade-off between present and future and intergenerational equity) when the risk premium takes into account the project's exposure to macroeconomic risk, i.e. in practice the correlation between the change in a project's sectoral benefits and

[6] French General Planning Commission (2005), *Le prix du temps et la décision publique*. [The cost of time and public decision-making.] *Révision du taux d'actualisation public [Revision of the public discount rate]*, report of the group of experts chaired by Daniel Lebègue, Paris, La Documentation française.

[7] Ramsey F. P. (1928), "A Mathematical Theory of Saving", *The Economic Journal*, vol. 38, no. 152, p. 543-559.

[8] Centre for Strategic Analysis (2011), *Le Calcul du risque dans les investissements publics [Calculating risk in public investment]*, report of the mission chaired by Christian Gollier.

[9] French General Commission for Strategy and Foresight (2013), *L'Évaluation socioéconomique des investissements publics [The socioeconomic evaluation of public investment],* report of the mission chaired by Émile Quinet, September.

economic growth, penalising pro-cyclical projects and valuing counter-cyclical projects.

The systemic risk premium, noted φ, which integrates both the volatility of the economy and the relative aversion of the community to risk, is such that the risk premium relating to a project is proportional, the coefficient of proportionality specific to the project being the sectoral β of the project (thus with β.φ for project risk premium).

This logic leads to the formula highlighted in the É. Quinet report (2013):

$$\rho = r_f + \beta.\varphi$$

where

r_f: reference discount rate

φ: macroeconomic systemic risk premium

β: elasticity of the net benefits of the project compared to GDP per head

If $\beta = 0$, r_f is the systemic "risk-free discount rate". The É. Quinet report (2013) thus recommended using a value of 2.5% for the reference discount rate and 2% for the risk premium, i.e.

$\rho = 2.5\% + \beta.2\%$ until 2070

and $\rho = 1.5\% + \beta.\ 3\%$ beyond.

It is these recommendations on the discount rate that have been in force since 2013. In practice, the risk premium is rarely differentiated and a single discount rate of 4.5% used.

The 2021 revision of the discount rate

Conducting project assessments with an incorrect discount rate may result in an inefficient allocation of public resources. An excessively low risk-free reference rate leads to an excess of investments; an excessively high risk-free reference rate, which reduces the expected long-term benefits relative to the costs to be borne in the short term, may lead to an investment deficit relative to the optimal level. A risk premium that is too low (and/or insufficiently differentiated depending on the projects) leads to a poor assessment of the connection between sectoral risks and collective risk. It does not sufficiently penalise the projects most exposed to a fall in GDP and does not sufficiently value insurance projects. In particular, the single rate of 4.5% used does not take into account the specific and countercyclical long-term nature of public health investments.

Since 2017[10], the Committee of Experts[11] on Methods for Socioeconomic Evaluation of Public investments, chaired by Roger Guesnerie, has conducted reflections[12] to update the

[10] See the minutes of the meeting of 8 June 2017 on the France Stratégie website.

[11] See the inaugural session of the Committee on the France Stratégie website.

[12] In July 2017, Roger Guesnerie and Christian Gollier drafted a joint note presenting a simplified analytical background in order to provide a common framework (note available on the France Stratégie website). A working group was set up and two internships were carried out in 2018 (Victoria Grimaud) and 2019 (Asmae Marhraoui) at France Stratégie under the management of Joël Maurice, Émile Quinet and Jincheng Ni. The Committee of Experts supported its work on the long series of real GDP per head of France from 1820 to 2016, prepared by Gilbert Cette *et al.* (Banque de France). It also organised a consultation and a vote, and issued its final opinion at the meeting on 24 June 2021.

recommendations of the É. Quinet (2013) report on risk and discount rates.

In the update system proposed by the Gollier (2011) and the É. Quinet reports (2013), economic growth expectations play a key role in both the risk-free reference discount rate and the average market risk premium. Stronger growth justifies a higher (risk-free) reference discount rate (asset effect). Greater uncertainty on GDP justifies a lower reference rate and a higher risk premium (precautionary effect).

The theoretical model sought is that of Ramsey[13] expanded to take into account the uncertainties of economic growth and the greatest or lesser impact of these uncertainties on the project. According to this model, the discount rate is given by the following formula:

$$\rho = r_f + \beta . \phi$$

where

$$r_f = \delta + \gamma . \mu - \frac{1}{2} . \gamma^2 . \sigma^2$$

$$\phi = \gamma . \sigma^2$$

δ: rate of time preference

γ: instant risk aversion

β: elasticity of the net benefits of the project compared to GDP per capita

μ: real growth rate of GDP per capita, as an annual average over the calculation period

σ: standard deviation (“volatility”) of the real growth rate of GDP per capita

The extended Ramsey formula (1928), which is at the heart of discount theory, is accurate from an analytical point of view if and only if the random variable of the GDP per capita growth rate follows a purely Gaussian probability law.

According to this formula, the reference discount rate with no risk r_f translates:

- the pure preference of economic agents for the present, i.e. a preference for immediate well-being over the same future well-being;
- the wealth effect, which, through an anticipation of economic growth, leads to less value being given to a gain in consumption tomorrow than the same gain today (the marginal utility of consumption is decreasing with the level of this consumption);
- a precautionary effect: the uncertainty of the growth in consumption leads to the postponement of part of the consumption to the future as a precautionary measure. This precautionary effect reduces the discount rate and gives more weight for the future.

By integrating future uncertainty, we obtain the wider Ramsey formula. The relationship between discount rates and time horizons is thus arbitrated by two effects: a wealth effect that encourages current generations to spend more and a precautionary effect that encourages them to spend less.

The risks of a public investment project are of two types: systemic risk dependent on the uncertainty of future economic growth and the specific non-systemic risk specific to the project. Taking into account systemic risks in the cost-benefit calculation of a project requires the inclusion of a systemic risk premium that can be seen as the expected additional profitability required to compensate for the collective risk of a project whose benefits are subject to the same uncertainties as GDP per capita.

[13] Ramsey F. P. (1928), “A Mathematical Theory of Saving”, *op. cit.*

The Committee of Experts wished to revise the public discount rate downwards by adopting a lower risk-free reference rate and maintaining a fairly high level of risk premium. A downward risk-free reference rate is in fact consistent with real interest rates that have fallen significantly and the lower growth potential of the French long-term economy (see COR (the French Pension Advisory Board) assumptions). Maintaining a fairly high risk premium incorporates likely macroeconomic shocks from long-term climate and/or health issues (as illustrated by the Covid crisis).

However, by applying the extended Ramsey formula, a very low systemic risk premium is obtained. The link between these two terms therefore conflicts with a theoretical difficulty because no level of risk aversion is capable, in a simplified model where the same parameter is used to measure agent risk aversion and the inter-temporel substitution coefficient, to report on low risk-free reference rates and high risk premiums[14].

Taking into account rare disasters

A justification for a high risk premium lies in the existence of extreme risks (rare disasters) whose low probability of occurring is not properly provided by a Gaussian law. The introduction of these extreme risks under the standard model is based on Barro's modelling (2006, 2009, 2011)[15] and its systemic risk assumptions. This approach makes it possible to build models and configure them, with values that can be justified given the past, so that they can generate both low risk-free interest rates and fairly high risk premiums (Barro and Tin, 2011).

The working group of the Committee of experts' explained the implicit process of the discount rate of the É. Quinet report (2013). The theoretical framework (see technical appendix) was renewed with the explicit consideration of rare disasters in Barro (2011)[16]. In this new theoretical framework, it is assumed that the random variable of the GDP per capita growth rate is the sum of two independent random variables: one supposed Gaussian and one non-Gaussian to represent rare disasters. The discount rate is the sum of the preference rate for the present and two components related respectively to each of these two independent variables.

The distribution of rare disasters is assumed to follow a Pareto's law of probability. This new theoretical framework requires eight parameters to determine the discount rate:

- the pure preference rate for the present;
- the relative risk aversion coefficient of the community;
- the expectation of the growth rate of GDP per capita in the future;
- the standard deviation ("volatility") of the growth rate of GDP per capita in the future;
- the correlation coefficient of the benefits or costs of the project with GDP per capita;
- the likelihood of an annual occurrence of a rare disaster;

[14] Taking risk aversion = 2 as recommended in the Gollier (2011) and the É. Quinet (2013) reports, we could only reach = 2% if we had the variance (σ^2) = 1% because $\phi = \gamma . \sigma^2 = 2.\ 1\% = 2\%$, i.e. a value significantly higher than historical GDP per capita findings in France which give a value of 0.374% for 1820-2016, 0.475% for 1913-2016 or 0.023% for 1973-2016.

[15] Barro R. J. (2006), "Rare disasters and asset markets in the twentieth century", *Quarterly Journal of Economics*, vol. 121, p. 823-866; Barro R. J. (2009), "Rare disasters, asset prices, and welfare costs", *American Economic Review*, vol. 99, P. 243-264; Barro R.J. and Jin T. (2011), "On the size distribution of macroeconomic disasters", Harvard University, February.

[16] Barro R.J. and Jin T. (2011), *op. cit*.

- the threshold of rare disasters (falling more than 10% of GDP per capita);
- the elasticity of Pareto's law.

This theoretical framework was applied on the basis of the aforementioned series of GDP per capita from 1800 to 2016 provided by Gilbert Cette *et al.*[17] (Banque de France). Some parameters obtained on data from the past of France have been reused to determine the new discount rate for the future.

The socioeconomic net present value (SE-NPV) of the project is obtained by applying the discount rate to the mathematical expectation of the net annual benefit of the project, which is also dependent on the elasticity of β compared to real GDP per head. This rate of change ν on the expectation of the net annual benefit of the project depends, in addition to β, on the statistical characteristics of future growth.

The new recommended rate

In applying this theoretical framework, the Committee of Experts adopted the principle of a risk-free reference rate consistent with the long-term growth assumptions of the low medium scenario resulting from the work of the Pension Advisory Board and maintaining a sufficiently high level of risk premium to make it possible to clearly differentiate the sensitivity of projects to economic shocks. In the opinion deliberated on 24 June 2021, the Committee of Experts makes the following recommendations:

- **using a discount rate equal to ρ = 1.2% + β.2% for the period from 2021 to 2070;**
- **the rate of change in mathematical hope for the benefit of a project is v = $\beta.1.15\ \% + \frac{\beta^2}{2}.0.9\ \%$;**
- **if it is unknown, it is proposed to proceed as if β were equal to 1, the rate elsewhere to be used is 3.2%, the rate of change in the mathematical expectation of benefit v is 1.6%.**

We can refer to the technical appendix to find out the values taken of the parameters that led to this result contained in the opinion of the Committee of Experts.

An operational supplement for standard cases will be drawn up to guide project leaders in the simultaneous use of the discount rate and the mathematical expectation of the benefit v.

The implementation of these recommendations implies the definition of the coefficients of "β" involved in the discount rate formula. The Committee of Experts has already provisionally set the values of this coefficient for the transport sector and civil engineering, in line with the corresponding values set out in the É. Quinet report (2013). For example, for transport[18]:

- Urban travellers : 1.1
- Regional travellers : 1.2
- Long-distance travellers : 1.7
- Freight rail : 1.4

However, these values must be updated to take into account the new economic conditions,

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[17] http://www.longtermproductivity.com/

[18] Report É. Quinet (2013), *op. cit.*, p. 82.

those which precisely led to a change in the discount rate, and to be adapted to future risks as can now be understood.

A working group "Estimate of the β" was set up for this. Its task will be to estimate these coefficients more accurately and appropriately for both transport and civil engineering sectors, and then to gradually extend the estimates to the other sectors. Its work will be published on an ongoing basis, after validation by the Committee of Experts. In the immediate future, for sectors for which no value has been formalized by the Committee of Experts, the Committee of Experts decided that a value of 1 should be assigned to this coefficient by default.

Of course, in the socioeconomic assessment, account must also be taken of the risks specific to the project, independent of macroeconomic growth, which are two-fold:

- risks that may - at least partially - be controlled by the project owner and which result, for example, from errors in the estimates of deadlines and costs (construction, maintenance, operation, environmental and health protection, etc., generally underestimated) or future pricing practices of the operator. The analysis should consider the measures that can be put in place to reduce these risks;
- the risks associated with the implementation of the evaluation relating to the estimates of the components of the SE-NPV: they may result from the use of unreliable data (poor data quality), inadequacies and imperfections of demand "models", or from the difficulty of predicting behavioural developments, changes in regulations and pricing rules, the emergence of new competitions, the obsolescence of technologies, etc.

The operational supplement "risk mapping"[19] presents the methods for taking these risks into account.

[19] Also available on the France Stratégie website.

Appendix 1

The theoretical framework, the calculation process and the results[20]

1. The theoretical framework

1.1. Formulations of the “socioeconomic net present value” (SE-NPV) and the discount rate ρ_t

Reference: Gollier-Guesnerie note of July 2017, “Discussion sur l’actualisation: un arrière-plan analytique” [Discussion on updating: an analytical background].

The socioeconomic net present value (SE-NPV) of the project measures the change in the mathematical expectation of the monetised collective utility that this project generates throughout its lifetime. The discount rate ρ_t to the mathematical expectation of the net annual benefit[21] A_t of the project. All monetised flows are expressed in euros for the year of update[22].

The collective utility over time “outside the project” is assumed in form[23]:

$$W = \sum_{t=0}^{t=+\infty} e^{-\delta.t}.P_t.E\frac{Y_t^{1-\gamma}-1}{1-\gamma}$$

where

δ: the pur preference for the present;

γ: risk aversion;

P_t: the population in year t;

Y_t: GDP per capita of year t.

The monetised value of the collective utility is obtained by dividing W by $Y_0^{-\gamma}$, i.e. by the usefulness of an additional 1 euro of GDP per capita of the year of discounting.

The annual net benefit A_t of the project is “small” compared to GDP, and it is assumed to depend on the GDP per capita of year t by the relationship:

$$A_t = \bar{A}_t.\left(\frac{Y_t}{Y_0}\right)^{\beta_t}$$

where

$\bar{A}_t$ is a scale factor;

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[20] The assumptions, deductions and calculations are specified in Maurice J. (2021) “Note justificative du 26 août 2021” [Supporting note dated 26 August 2021] (available on the France Stratégie website).

[21] Profit flow of the project less cost flow, during year t.

[22] See France Stratégie/General Treasury Directorate (2017), *Guide de l’évaluation socioéconomique des investissements publics [Guide to the socioeconomic evaluation of public investment]*, December, page 31: “The discount year is set for 2015 by the Committee of Experts for socioeconomic assessments carried out between 2017 and 2022”.

[23] The monetised value of the collective utility is obtained by dividing W by $Y_0^{-\gamma}$, i.e. by the usefulness of an additional 1 euro of GDP per capita of the year of discounting.

βt is the elasticity of At compared to Yt: it depends on the content24 of At.

In particular, we will seek to examine the influence that this elasticity β_t has on the discount rate and on the SE-NPV of the project.

We consider the annual variable $z_t = ln\frac{Y_t}{Y_{t-1}}$, in other words $z_t = \ln(1 + g_t)$ where g_t is the annual growth rate of Y_t. It is assumed that, each year t, z_t is a random variable and that, when t varies, these variables z_t are independent and identically distributed *(iid),* of type z.

Let's call *SE-NPV(A_t)* the contribution of the net annual benefit A_t to the SE-NPV. With the above assumptions, the following fundamental expression is obtained:

$$SE\text{-}NPV(A_t) = e^{-\delta.t}.\bar{A}_t.[Ee^{(\beta_t-\gamma).z}]^t$$

The discount rate ρ_t, applicable to the annual net benefit A_t is then defined by the following relationship:

$$SE\text{-}NPV\ (A_t) = e^{-\rho_t.t}.\bar{A}_t.[Ee^{\beta_t.z}]^t$$

We define the following two functions:

$$\tau_t = -\ lnEe^{(\beta_t-\gamma).z}$$

$$\nu_t = lnEe^{\beta_t.z}$$

We then have the identity:

$$\rho_t = \delta + \tau_t + \nu_t$$

ν_t can be interpreted as "the rate of change of the mathematical expectation of the net annual benefit A_t" of the project, under the influence of the elasticity β_t compared to the GDP per capita. τ_t can be interpreted as the "overall discount rate of the *SE-NPV (A_t)* component of added value", under the influence of elasticity β_t in relation to GDP per capita. This rate encompasses the two effects on *SE-NPV* (A_t) due to uncertainties concerning GDP per capita: the effect that passes through the discount rate and the effect that passes through the mathematical expectation of the net annual benefit of the project.

We will endeavour to examine the influence that this elasticity β_t exerts on these two components τ_t and ν_t of the discount rate ρ_t.

Case of the existence of "risks specific" to the project: β_t = 0

If the project also involves "own risks" that are independent of Y_t, they are subject to the above relationships, for which β_t = 0. We would then have ν_t = 0 and therefore:

$$\rho_t = \delta + \tau_t$$

where $\tau_t = -\ln Ee^{-\gamma.z}$

[24] For example: investment, operation, etc.

This share of the benefits exposed to own risks would need to be updated by applying the so-called "r_{ft} macroeconomic risk-free" rate, thus given by: $r_{ft} = \delta - \ln Ee^{-\gamma.z}$

Special case $\beta_t = \gamma$

We would then have $\tau_t = 0$ and consequently:

$$\rho_t = \delta + \nu_t$$

where $\nu_t = \ln Ee^{\gamma.z}$

Based on the assumptions introduced successively above, it is possible to determine the unknowns sought, provided that the probability law of the random variable Y_t is available. It is then convenient to determine the two rates τ_t and ν_t, separately then the discount rate ρ_t as being their sum, increased by δ.

In order to simplify the ratings in the following, the index t is ignored, but it should not be forgotten that, every year t, the elasticity must in principle be appropriate to the content of the annual net benefit *A.*

1.2. Law of probability of the change in real GDP per capita: Gaussian law and the law of rare disasters

Random variable z is assumed to be the sum of two random independent variables:

$$z = za + zb$$

where

za is assumed to be Gaussian

zb is supposed to represent rare disasters; it is non-Gaussian.

Since the *za* and zb variables are assumed to be independent, so the following equations apply:

- the average (the mathematical expectation, or first-order cumulant) of the sum is the sum of the averages:

  $$k_1 = ka_1 + kb_1$$

- the variance (or second-order cumulant) of the sum is the sum of the variances:

  $$k_2 = ka_2 + kb_2$$

- the discount rate due to the sum is obtained by successively calculating, as a function of the variable β, highlighted here:

$$\tau_{a+b}(\beta) = \tau a(\beta) + \tau b(\beta)$$

$$\nu_{a+b}(\beta) = \nu a(\beta) + \nu b(\beta)$$

$$\rho_{a+b}(\beta) = \delta + \tau_{a+b}(\beta) + \nu_{a+b}(\beta)$$

The equivalent expression is deduced from this:

$$\rho_{a+b}(\beta) = \delta + \rho a(\beta) + \rho b(\beta)$$

where

$\rho a(\beta) = \tau a(\beta) + \nu a(\beta)$ is the component of $\rho_{a+b}(\beta)$ due to za

$\rho b(\beta) = \tau b(\beta) + \nu b(\beta)$ is the component of $\rho_{a+b}(\beta)$ due to zb

Since the random variable *za* is assumed to be Gaussian, the component noted $\rho a(\beta)$ of the discount rate relative to za is then given by the standard relationship, a linear function of β, because:

$$\tau a(\beta) = -[(\beta - \gamma).ka_1 + \frac{(\beta-\gamma)^2}{2}.ka_2] \quad \text{(which is of the second order in } \beta)$$

$$\nu a(\beta) = \beta.ka_1 + \frac{\beta^2}{2}.ka_2 \quad \text{(which is of the second order in } \beta)$$

$$\rho a(\beta) = \tau a(\beta) + \nu a(\beta) = (\gamma.ka_1 - \frac{\gamma^2}{2}.ka_2) + \beta.(\gamma.ka_2) \quad \text{(which is of the first order in } \beta)$$

The component denoted $\rho b(\beta)$ of the discount rate relating to zb, which is supposed to represent rare disasters, depends on the probability distribution postulated for zb. zb is supposed to follow a law of probability of Pareto of parameters $(p, \varepsilon_0, \alpha)$ as follows Barro (2011)[25] with:

p: annual probability of occurrence of a rare disaster

ε_0: threshold for rare disasters: any disaster causing a fall in real GDP per capita of an absolute value equal to or greater than ε_0 is classified as a rare disaster.

α: elasticity (positive) of Pareto's law

The following wording is then obtained:

$$\tau b(\beta) = -\ln(1 - p + p.\frac{\alpha}{\beta-\gamma+\alpha}.e^{(\beta-\gamma).(-\varepsilon_0)})$$

$$\nu b(\beta) = \ln(1 - p + p.\frac{\alpha}{\beta+\alpha}.e^{\beta.(-\varepsilon_0)})$$

$$\rho b(\beta) = \tau b(\beta) + \nu b = -\ln\left(1 - p + p.\frac{\alpha}{\beta - \gamma + \alpha}.e^{(\beta-\gamma).(-\varepsilon_0)}\right) + \ln(1 - p + p.\frac{\alpha}{\beta + \alpha}.e^{\beta.(-\varepsilon_0)})$$

Contrary to the component $\rho a(\beta)$, the component $\rho b(\beta)$ of the discount rate $\rho_{a+b}(\beta)$ is non-linear in β.

2. The calculation process

The above analytical formulas depend on the following seven parameters:

- the utility function: δ, γ
- anticipated changes in real GDP per capita: k_1, k_2
- the law of probability of rare disasters: p, ε_0, α

[25] Barro R. J. and Jin T. (2011), *op. cit*. Barro and Tin successively examine the hypothesis of a single Pareto law and that of a double Pareto's law. We have taken this at this stage at the first hypothesis.

If we give ourselves the values of these seven parameters of the model, these formulas make it possible to calculate the exact values of $\tau_{a+b}(\beta)$, $\nu_{a+b}(\beta)$ et $\rho_{a+b}(\beta)$ in accordance with β, which are varied, for example, from -1 to +2.

On the basis of these calculated values, approximations are sought to represent:

- the discount rate in a simplified form: $\rho(\beta) = rf + \beta.\phi$
- the rate of change in the expected net annual benefit in a simplified form $\nu(\beta) = \beta.k_1 + \frac{\beta^2}{2}.w$,
- while minimising errors in the calculation of the SE-NPV (At), which amounts to minimising errors in the overall discount rate compared to its analytical $\tau_{a+b}(\beta)$.

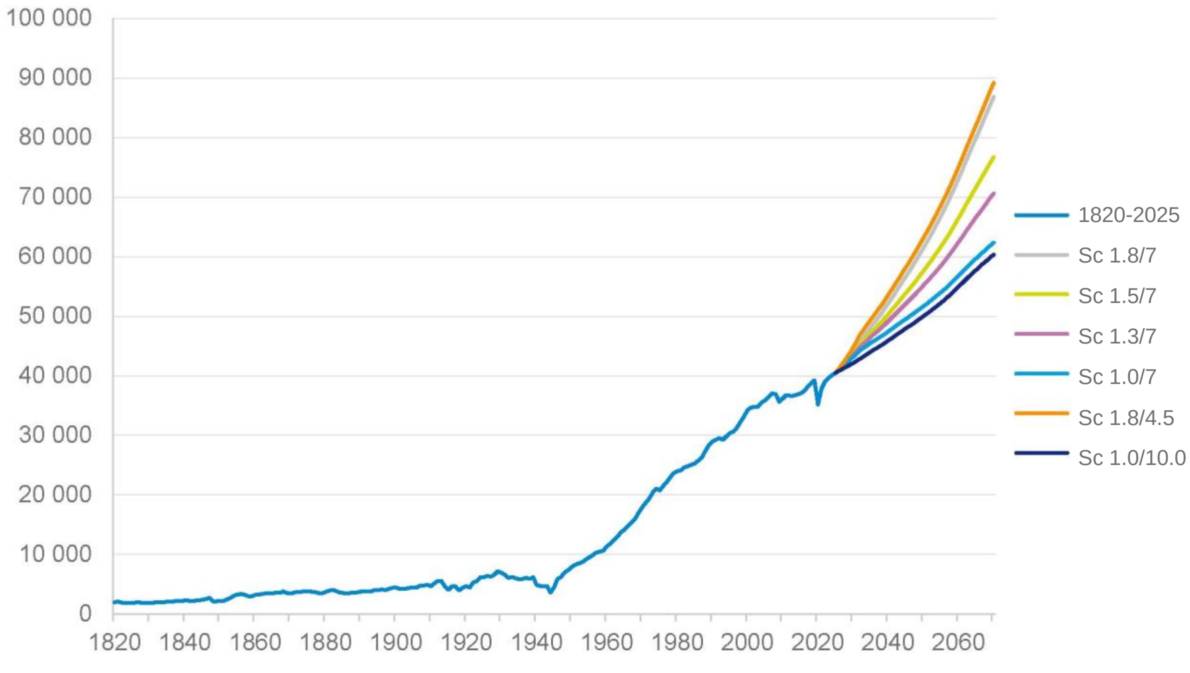
We perform the parabolic approximation $\tau_p(\beta)$ of the function $\tau_{a+b}(\beta)$ and then decompose $\tau_p(\beta)$ into $\rho_p(\beta)$ linear and $\nu_p(\beta)$ parabolic.

3. The values of the parameters and the result

3.1. COR 2020 macroeconomic scenarios

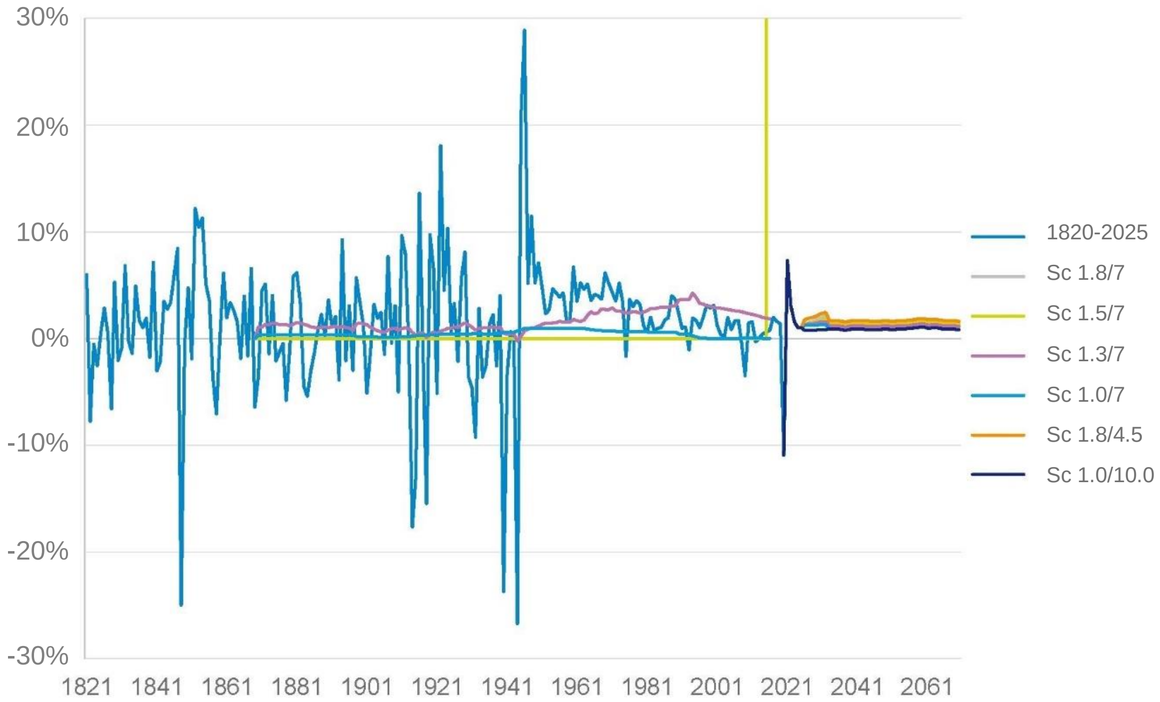
The following two graphs show the different scenarios for the growth of GDP per capita based on data from the COR 2020. Following the Covid pandemic in early 2020, these six scenarios differ from 2026, with the five years 2020 to 2025 being assumed the post-Covid transition period.

Chart 1 - Changes in GDP per capita (COR scenarios)



Sources: G. Cette et al. *(1820-2016) and Insee + COR scenarios (2017-2070)*

Graph 2 - Change in LN delta (GDP per capita)



Sources: G. Cette et al. *(1820-2016) and Insee + COR scenarios (2017-2070)*

We obtain the mean k_1 of the random variable z_t over the period 2019-2070 for each scenario.

Table 1 - Statistics of COR macroeconomic scenarios

No.	Type	k_1	Standard deviation	Variance
1	Sc 1.8/4.5	1.6103%	1.98%	0.039%
2	Sc 1.8/7	1.5581%	1.97%	0.039%
3	Sc 1.5/7	1.3146%	1.95%	0.038%
4	**Sc 1.3/7**	**1.1524%**	1.94%	0.038%
5	Sc 1.0/7	0.9084%	1.94%	0.037%
6	Sc 1.0/10.0	0.8440%	1.93%	0.037%

Source: COR 2020, calculation Joël Maurice and Jincheng Ni

The low average COR 2020 scenario was used - i.e., in the previous table, no. 4 (scenario 1.3/7) with the assumption of 1.3% of annual growth in labour productivity and 7% of the unemployment rate - in accordance with the decision of the Committee of Experts of 10 October 2017 (see “The configuration of the reference scenario”, Supplement A1 to the *Guide de l’évaluation socioéconomique des investiisements publics* [Guide to the socioeconomic evaluation of public investment]).

The variance noted k_2 of the random variable z_t is based on the historical series of real GDP per capita in ppa established by Cette *et alii* (Banque de France).

Table 2 - Variances according to historical periods

Period	Variance
1820-2016	0.374%
1900-2016	0.445%
1913-2016	**0.475%**
1939-2016	0.391%
1947-2016	0.052%
1973-2016	0.023%

Source: G. Cette, calculation Joël Maurice and Jincheng Ni

3.2. The values of the parameters and the result

The values of the seven parameters used in the calibration of the discount rate are as follows:

- the GDP per capita growth rate is 1.1524%, corresponding to the low average scenario of the COR 2020;
- the preference rate for the present is 0.435, a decision taken after a vote by the Committee of experts;
- the variance of GDP per capita is 0.475, corresponding to the historical value for the period 1913-2016 for France. In fact, the low average scenario for 2070 in the COR 2020 has a variance of 0.038%. As this variance is extremely low, which according to the General Treasury Directorate is linked to the estimation period of the macroeconomic model used, the Committee of Experts proposes substituting the maximum variance of the French historical series (G. Cette) over the period from 1913 to 2016, which saw the major disasters of modern history (14-18 war, Great Depression, 1939-1945 war, oil crisis, financial crisis, etc.);
- risk aversion γ is 2.478. This value is greater than 2, which was that of the Gollier (2011) and the É. Quinet reports (2013). Given the Covid pandemic, which increases uncertainty about the future, and to keep the risk premium level at 2%, we need to increase risk aversion γ to 2.478;
- the annual probability of occurrence of a rare disaster is 0.0383, i.e. every 26 years, corresponding to the “single Pareto distribution” hypothesis in Barro (2011);
- the threshold for rare disasters is set at a 10% fall in real GDP per capita, corresponding to the “single Pareto law” hypothesis in Barro (2011);
- the (positive) elasticity of the Pareto law is 6.86, corresponding to the “single Pareto law” hypothesis in Barro (2011).

With these values of the parameters, the result[26] is as follows:

[26] See Maurice J. (2021), “Note justificative du 26 août 2021” [Memorandum of 26 August 2021].

Table 3 - Discount rate according to β

Results as a function of β		
β	ρ	ν
-1.0	-1.132	-0.889
-0.5	0.246	-0.514
0.0	1.318	0.000
0.5	2.182	0.633
1.0	2.900	1.371
1.5	3.513	2.202
2.0	4.047	3.118

Source: calculation of Joël Maurice

By using the usual formulation and looking for the coefficients that give the best approximation of the SE-NPV, we arrive at the following expressions[27]:

– the discount rate in simplified form:

$\rho(\beta) = rf + \beta.\phi = 1.2\ \% + \beta.2\ \%$

– the rate of change in the expected net annual benefit in simplified form:

$\nu(\beta) = = \beta.1.15\ \% + \frac{\beta^2}{2}.0.9\ \%$

[27] The calculation process is specified in Maurice J. (2021), *op. cit.*

Appendix 2

Opinion and recommendations of the Committee of Experts of 24 June 2021

France Stratégie, General Secretariat for Investment

Committee of Experts on Methods for the Socioeconomic Evaluation of Public investments

The Chairman,
Roger Guesnerie

Paris, 1st July 2021

The revision of the discount rate

Opinion and recommendations of the Committee of Experts on the socioeconomic calculation of public investment

Based on the request made in the mission letter sent by the Commissioner General of France Stratégie and the Secretary General for Investment on 21 February 2019, the Committee has conducted a review to update the recommendations of the Quinet report (2013) on risk and discount rates.

The Émile Quinet report (September 2013) recommended the following values for the discount rate on public investments:

$$\boldsymbol{\rho} = 2.5\% + \beta.\ 2\% \text{ up to 2070}$$

$$\text{and } \rho = 1.5\% + \beta.\ 3\% \text{ beyond}$$

where β is the elasticity of the project's annual benefits compared to GDP per capita.

The issue has given rise to a great deal of work and debate, resulting in an opinion deliberated on 24 June 2021, in which the Committee makes the following recommendations:

- **Use a discount rate equal to ρ = 1.2% + β. 2%**
- To calculate the mathematical expectation of a project's benefit, the coefficient to be applied is $\boldsymbol{\nu(\beta)} = = \boldsymbol{\beta.1.15\ \% + \frac{\beta^2}{2}.0.9}\ \%$
- If β is unknown, it is proposed to proceed as if β were equal to 1. The rate p to be used is then 3.2% and the rate v is 1.6%. We have ρ - v = 1.6%.
- More needs to be done to explain and use the formulas, and further thought needs to be given to the discount rate after 2070.

The Committee suggests that these recommendations be the subject of instructions to public project owners, supplementing the "Guide de l'évaluation socioéconomique des investissements publics" [Guide to the SocioEconomic Evaluation of Public Investments].

Roger GUESNERIE

Appendix 3

Response from the Commissioner General of France Stratégie

PREMIER MINISTRE
Liberté
Égalité
Fraternité

FRANCE STRATÉGIE
ÉVALUER. ANTICIPER. DÉBATTRE. PROPOSER.

The Commissioner general

Paris, 30 September 2021

Subject: Guide to the socioeconomic evaluation of public investments in France, revision of the discount rate

Mr President, dear Roger Guesnerie,

I have received your progress report with the opinion and recommendations of the Committee of Experts on the revision of the discount rate. Thank you. This is the culmination of a very important work, carried out from early 2019 to mid-2021, and leads in a period of great uncertainty.

Socioeconomic assessment is one of the main tools available to the State to assess the relevance of a public investment project for the community. The discount rate plays a central role in this evaluation. Conducting socioeconomic assessments of public projects with an incorrect discount rate may result in an inefficient allocation of public resources. The issue of the revision of the public discount rate set in 2013 is all the more crucial since two factors of uncertainty have since taken on an increased dimension: those relating to climate, those relating to health.

After several years of reflections and numerous debates within your committee, you resulted in the opinion and recommendations adopted at the meeting of 24 June 2021. I congratulate you on this achievement and thank all those who worked with you on this project.

Your committee revised the public discount rate used in 2013 downwards, retaining a lower risk-free rate, and maintaining a fairly high level of risk premium. You argue that this downward revision of the risk-free rate is consistent with real interest rates that have fallen sharply and with the growth potential of the French long-term economy (see COR assumptions). Maintaining a fairly high risk premium incorporates likely macroeconomic shocks reflecting climate and/or health issues (as illustrated by the current COVID crisis) in the long term and makes it possible to differentiate the sensitivity of projects to economic shocks.

…/…

Mr Roger Guesnerie
Honorary President of Paris School of Economics
48 Boulevard Jourdan
75014 Paris

I recommend that you rapidly draw up an operational supplement to the *Guide to Socioeconomic Evaluation of Public Investments in France* so that this new public discount rate can be applied by sectoral project developers. Your working group on estimating sectoral betas must also continue its work in order to provide robust and shared sectoral beta values as soon as possible, with a view to differentiating the discount rate by sector.

Yours faithfully,

Gilles de Margerie

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