

## Conference on Discount rate in the selection of public projects

### Questions around Discount rate

motivated by my experience in financial interest rate

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## Brief presentation

### I apologize

- To have no macroeconomic culture
- In particular in the evaluation of long term investment public project
- to use not exactly your vocabulary, but a vocabulary inspired by the market

### A long experience in financial market

- A responsible of a well-known Master Program
- As a contributor to the abstract theory of interest rate, change of numeraire, yields curve dynamics
- and ten years of consulting in Model validation for the IRS products
- A large academic culture in portfolio/consumption optimization

### A look from the outside

## About the Use of the Ramsey Rule

How conciliate this rule with the complexity of modeling so distant future

- Why a constant discount rate even (slightly modified) for such a long horizon
- How to explain a such dispersion in the solution
- For a "probabilist" as me, to have such a certainty on the preference rate for the present  $\delta$  is very surprising, but I understand the idea of "prescriptive" approach
- The uncertainty is timidly introduced on the consumption rate...

First main question : What is a **wrong** discount rate ?

- What is the main question that we have to solve in the context of public project analysis with the **discounting**
- Given the ambiguity on the level of the rate, which strategy can be developed to increase the "**robustness**" of the NPV of the project ?
- If the NPV a good indicator ?

## Some more technical interrogations

### Some theoretical remarks valid for different universes

- given the rate  $\delta$ , the basic tools are a) the dynamics of the consumption rate  $c_t$  issued from  $c_0$ , and b) the marginal utility of the consumption  $u'(c_t)$ .
- Using small perturbation method, the links between discounting rate and growth is obtained by equaling the derivative  $R_t = \delta - \frac{1}{T} \ln E[u'(c_t)]/u'(c_0)$
- The Ramsey rule with constant parameters is obtained by assuming that the log of the dynamics of  $c_t$  is given by a Brownian motion  $(\mu, \sigma^2)$  issued from  $\ln(c_0)$  and a power utility  $u'(c) = c^{-1/\gamma}$  ( $\gamma$  is the risk aversion coefficient)

### Pricing kernel

- In finance  $e^{-\delta t} u'(c_t)/u'(c_0) = Y_t(c_0)$  is called **stochastic "pricing kernel"**
- the NPV of a cash flow  $B_t$  at time  $t$  is given by  $\mathbb{E}[B_t \cdot Y_t(c_0)]$ .
- In incomplete market, this marginal utility price holds only for **small cash flow**. For a large cash flow, "second order" premium is introduced.

## At the equilibrium

### Some consequences

- For no power utility and  $c_0$ - linear consumption rate,  $R_{0,t}$  is depending of  $c_0$  or from the wealth via some budget constraint.
- The theory said that the consumption has to be chosen optimally in an economy at the equilibrium.
- Only in very limited case the optimal solution is linear from its initial condition.
- Moreover when optimizing the equilibrium strategy for agents with different risk aversion, the utility of the representative agent is no longer a power function.

### Financial theory of Interest rates

- May be used to test different theoretical models,
- for instance, many yield curves are decreasing in the long term, due the

volatility of the rate

## Forward or backward point of views

### At the equilibrium

- The optimisation is starting from the horizon
- and is going to the present by retropropagation
- Often complex to solve
- Principle of **Time consistency**

### The forward Point of view

- The optimal consumption is given and diffusing to now to the future
- Allows some flexibility to integrate new knowledge

### How to use these remarks

- In defining some indicators concerning the more important risks(= parameters)
- in addition to the use of the classical Ramsey rule

## Conclusion

### Very complex task

- Complexity induces non linearity, introducing large bias in a priori too simple models
- The simplicity can only be used after identification of the main risk factors
- Simulation tools have become more efficient

### Adaptative Criterium

- To Integrate that decision criterion has to become more adaptative
- To deal with the uncertainty of "climate model, or long term risk" and its impact on the discount rate, develop idea about **vigilance**, to detect in advance the future evolution

**Thank you for your attention**